

# Comet Mission

## 2 Comet properties

```
a := 3035;
```

```
b := 2520;
```

```
c := 6050;
```

$$f[x_, y_] := \sqrt{1 - \left(\frac{6050 x}{3035}\right)^2 - \left(\frac{6050 y}{2520}\right)^2};$$

```
ContourPlot[f[x, y], {x, -4000, 4000}, {y, -3000, 3000},
```

```
PlotRange → All, BoxRatios → { $\frac{a}{c}$ ,  $\frac{b}{c}$ , 1}, Mesh → None, ContourStyle → {Gray},
```

```
AxesLabel → {"x", "y"}, PlotLabel → Style["Nucleus", Large]]
```

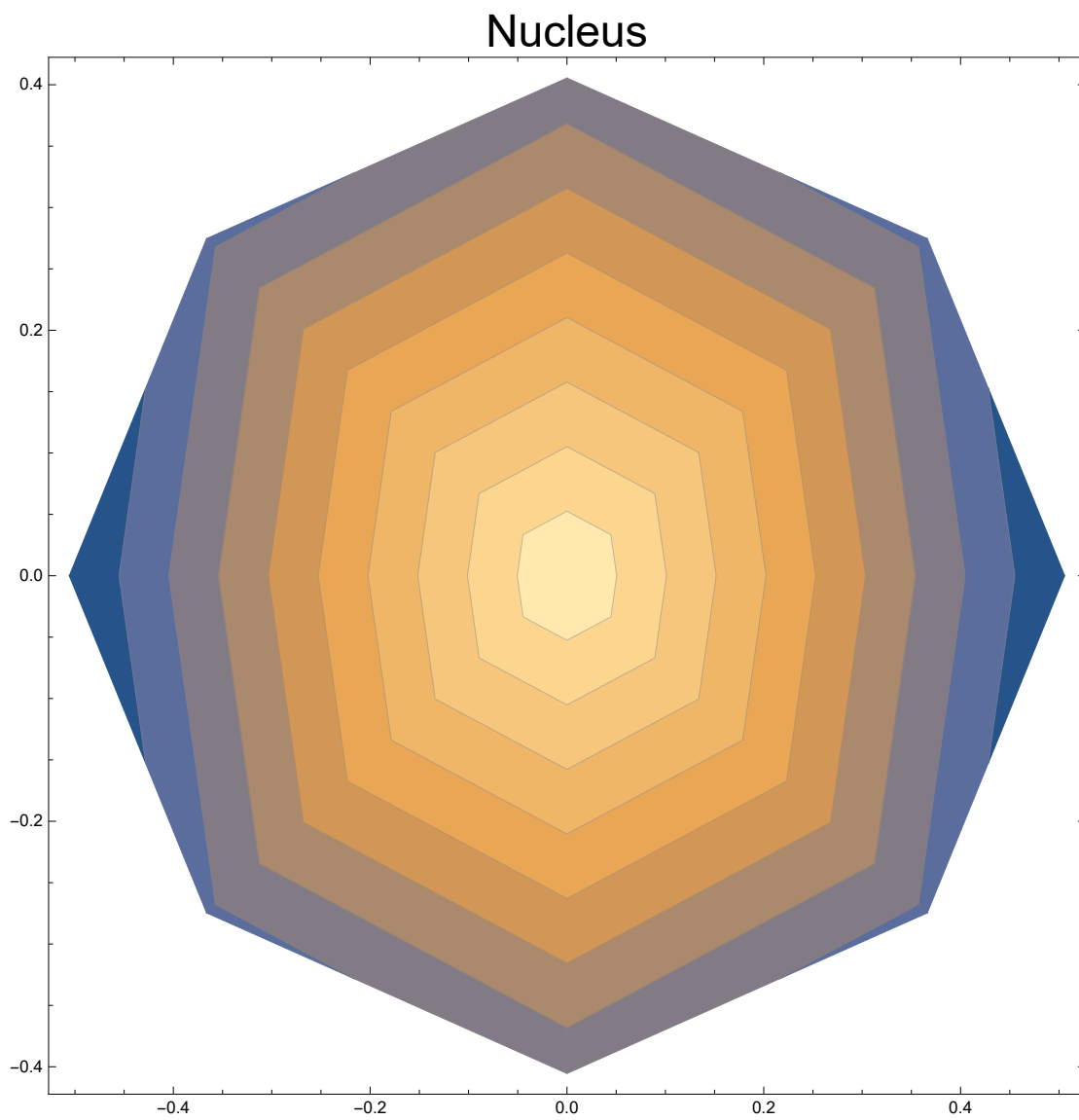
```
ContourPlot3D[( $\frac{x}{a}$ )2 + ( $\frac{y}{b}$ )2 + ( $\frac{z}{c}$ )2 == 1, {x, -a, a}, {y, -b, b}, {z, -c, c},
```

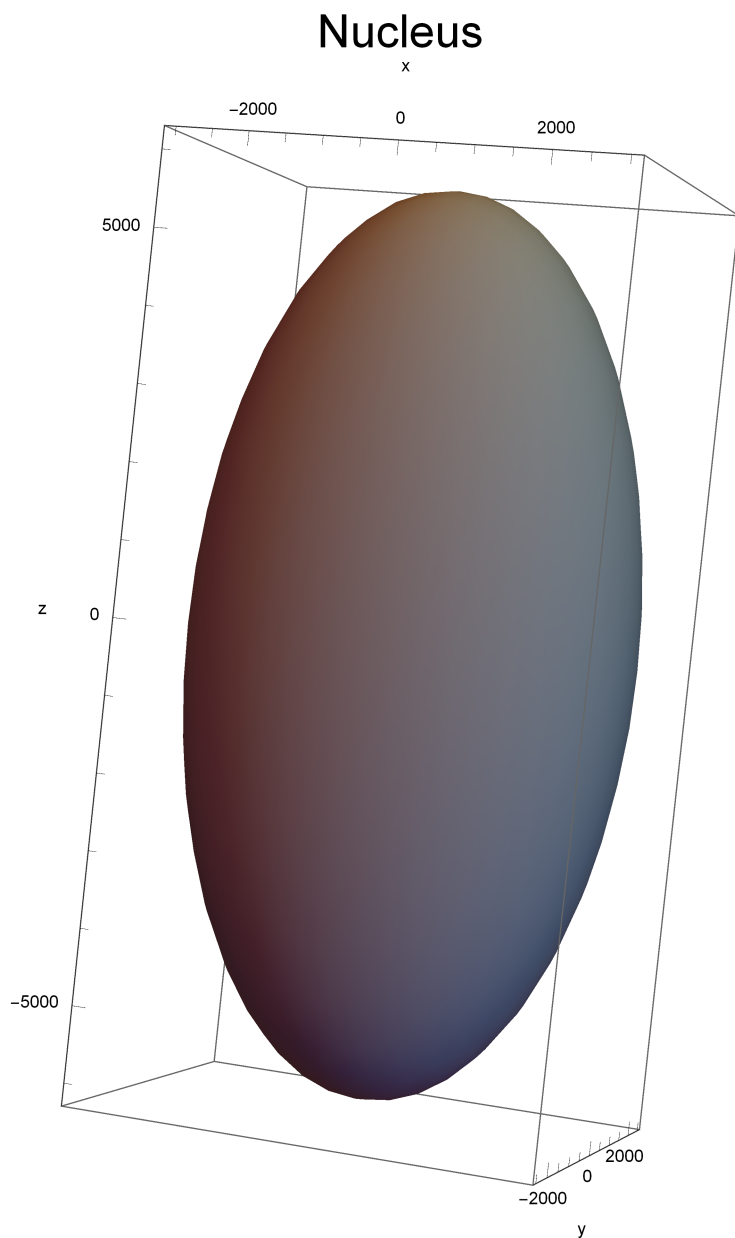
```
PlotRange → All, BoxRatios → {a / c, b / c, 1}, Mesh → None, ContourStyle → {Gray},
```

```
AxesLabel → {"x", "y", "z"}, PlotLabel → Style["Nucleus", Large]]
```

```
fzx := D[f[x, y], {x, 1}]
```

```
fzy := D[f[x, y], {y, 1}]
```





```

Clear[a, b, c, x, y, z]
a := 3035;
b := 2520;
c := 6050;
x := a * ρ * Sin[ϕ] * Cos[θ];
y := b * ρ * Sin[ϕ] * Sin[θ];
z := c * ρ * Cos[ϕ];
Mass = ∫0π ∫02π ∫01 400 (a * b * c) ρ2 * Sin[ϕ] dρ dθ dϕ
24 678 192 000 000 π

```

$$I_x = \int_0^\pi \int_0^{2\pi} \int_0^1 400 (y^2 + z^2) \rho^2 \sin[\phi] \, d\rho \, d\theta \, d\phi$$

$$\frac{13\,744\,928\,000 \pi}{3}$$

$$I_y = \int_0^\pi \int_0^{2\pi} \int_0^1 400 (x^2 + z^2) \rho^2 \sin[\phi] \, d\rho \, d\theta \, d\phi$$

$$\frac{14\,660\,392\,000 \pi}{3}$$

$$I_z = \int_0^\pi \int_0^{2\pi} \int_0^1 400 (x^2 + y^2) \rho^2 \sin[\phi] \, d\rho \, d\theta \, d\phi$$

$$\frac{4\,979\,720\,000 \pi}{3}$$

**a := 3035;**

**b := 2520;**

**c := 6050;**

**SA = NIntegrate[**  
 $(\sin[\theta]) \sqrt{(a^2 * b^2 * \cos[\theta]^2) + (b^2 * c^2 * \sin[\theta]^2 * \cos[\phi]^2) + (a^2 * c^2 * \sin[\theta]^2 * \sin[\phi]^2)},$   
 $\{\theta, 0, \pi\}, \{\phi, 0, 2\pi\}]$

**1.78648 × 10<sup>8</sup>**

### 3 Multipole expansion

```

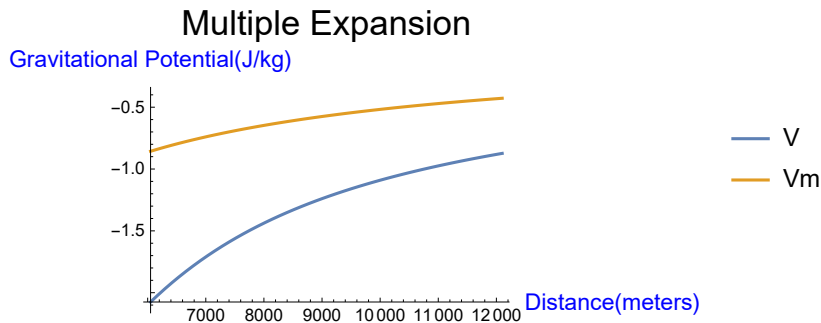
Clear[a, b, c, G, Mass]
a := 3035;
b := 2520;
c := 6050;
G := 6.67408 (10^(-11));
Mass :=  $\int_0^\pi \int_0^{2\pi} \int_0^1 400 (a * b * c) \rho^2 * \text{Sin}[\phi] \, d\rho \, d\theta \, d\phi$ ;
p[ξ_, η_, ζ_] := 400;

V[x_?NumericQ, y_?NumericQ, z_?NumericQ] :=
  -NIntegrate[ $\frac{G * p[\xi, \eta, \zeta]}{\sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}}$ , {ξ, -a, a}, {η, - $\sqrt{b^2 * (1 - (x^2) / (a^2))}$ ,
 $\sqrt{b^2 * (1 - (x^2) / (a^2))}$ }, {ζ, - $\sqrt{c^2 - ((x^2 c^2) / (a^2)) - ((y^2 c^2) / (b^2))}$ ,
 $\sqrt{c^2 - ((x^2 c^2) / (a^2)) - ((y^2 c^2) / (b^2))}$ }]];

 $\frac{-G * \text{Mass}}{\sqrt{x^2 + y^2 + z^2}} - \frac{G}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} * \text{Integrate}[$ 
  ( $\xi^2 + \eta^2 + \zeta^2$ ) * ( $3 * \frac{1}{2} * (3 * ((x * \xi + y * \eta + z * \zeta)^2) / ((x^2 + y^2 + z^2) * (\xi^2 + \eta^2 + \zeta^2)) - 1)$ ),
  {ξ, -a, a}, {η, - $\sqrt{b^2 - ((\xi^2 b^2) / (a^2))}$ ,  $\sqrt{b^2 - ((\xi^2 b^2) / (a^2))}$ },
  {ζ, - $\sqrt{c^2 - ((\xi^2 c^2) / (a^2)) - ((\eta^2 c^2) / (b^2))}$ ,
 $\sqrt{c^2 - ((\xi^2 c^2) / (a^2)) - ((\eta^2 c^2) / (b^2))}$ }]
H[x_, y_, z_] :=  $\frac{97.01879843294859 (981\,218 x^2 + 1\,324\,517 y^2 - 2\,305\,735 z^2)}{(x^2 + y^2 + z^2)^{5/2}} -$ 
 $\frac{5174.3359164239255}{\sqrt{x^2 + y^2 + z^2}};$ 

P = Plot[{V[0, 0, z], H[0, 0, z]}, {z, c, 2 c},
  PlotLabel → Style["Multiple Expansion", FontSize → 18],
  PlotLegends → {"V", "Vm"}, AxesLabel → {Style["Distance (meters)", Medium, Blue],
  Style["Gravitational Potential (J/kg)", Medium, Blue]}}]
 $\frac{97.0188 (981\,218 x^2 + 1\,324\,517 y^2 - 2\,305\,735 z^2)}{(x^2 + y^2 + z^2)^{5/2}} - \frac{5174.34}{\sqrt{x^2 + y^2 + z^2}}$ 

```



## 4 Landing on the comet

```

Clear[t, r1, ro, a, b, c, H, dotproduct]
a := 3035;
b := 2520;
c := 6050;
Mlander := 105;
r1[t_] :=
  {-7000 + 0.001 t + 3 * 10-6 * t2, -5000 + 0.001 t + 2 * 10-6 * t2, 10000 - 0.05 t - 2 * 10-6 * t2};
H[t_] := (97.01879843294859` (981 218 (-7000 + 0.001 t + 3 * 10-6 * t2)2 +
  1 324 517 (-5000 + 0.001 t + 2 * 10-6 * t2)2 - 2 305 735 (10000 - 0.05 t - 2 * 10-6 * t2)2)) /
  ((-7000 + 0.001 t + 3 * 10-6 * t2)2 + (-5000 + 0.001 t + 2 * 10-6 * t2)2 +
  (10000 - 0.05 t - 2 * 10-6 * t2)2)5/2 -
  5174.3359164239255` / (Sqrt((-7000 + 0.001 t + 3 * 10-6 * t2)2 +
  (-5000 + 0.001 t + 2 * 10-6 * t2)2 + (10000 - 0.05 t - 2 * 10-6 * t2)2));
(*landing time*)
FindRoot[{( (-7000 + 0.001 t + 3 * 10-6 * t2)2 / a +
  (-5000 + 0.001 t + 2 * 10-6 * t2)2 / b + (10000 - 0.05 t - 2 * 10-6 * t2)2 / c == 1}, {t, 0, 40000}]
v[t_] := Sqrt[r1'[t].r1'[t];
(*initial velocity*)
v[0]
(*initial force on the lander due to gravity*)
gradV[t_] := H'[t];
FonLander[t_] := -(Mlander) * gradV[t]
FonLander[0]
(*velocity at impact*)
v[42684.8]
H[42684.8]
dotproduct = FonLander[t].r1'[t];

WorkonLander = Integrate[dotproduct, {t, 0, 42684.8}];

```

```

WorkonLander2 = - (Mlander) * (H[42684.8] - H[0])

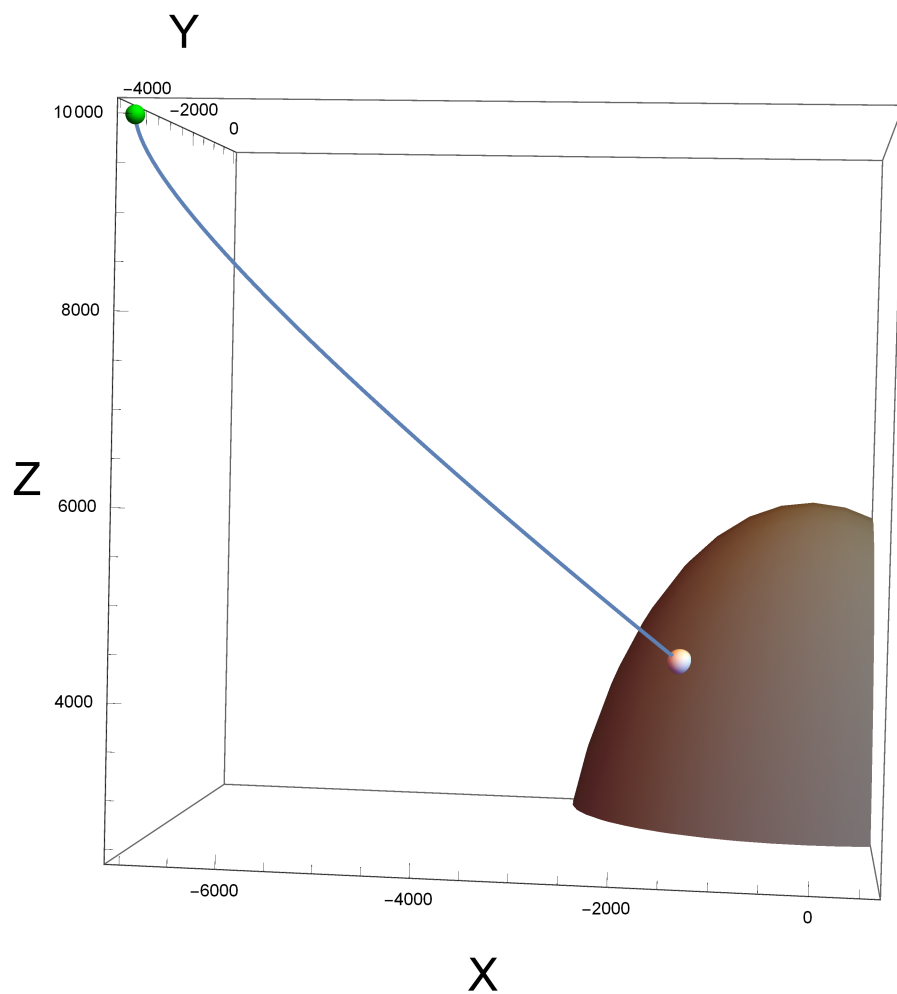
Kinetic[t_] = .5 * Mlander * (r1'[t]^2);
Potential[t_] = Mlander * H[t];
Kinetic[42684.8] + Potential[42684.8] - Kinetic[0] - Potential[0]
Potential[42684.8]
velocityatimpact = Sqrt[Abs[Potential[42684.8]] / (.5 * Mlander)]

Lander = ParametricPlot3D[{r1[t]}, {t, 0, 50000}, PlotLabel -> "Lander Trajectory",
  AxesLabel -> {
    Style["X", Large],
    Style["Y", Large],
    Style["Z", Large]}, SphericalRegion -> True,
  ImageSize -> Large];
Comet = ContourPlot3D[(X/a)^2 + (Y/b)^2 + (Z/c)^2 == 1, {x, -a, a},
  {y, -b, b}, {z, -c, c}, PlotRange -> All, BoxRatios -> {a/c, b/c, 1},
  Mesh -> None, ContourStyle -> {Gray}, AxesLabel -> {"x", "y", "z"}];
ReleaseLocation = Graphics3D[{Green, Sphere[r1[0], 100]}];
LandingLocation = Graphics3D[{White, Sphere[r1[42684.8], 150]}];
Show[Lander, ReleaseLocation, Comet, LandingLocation]
{t -> 42684.8}

0.05002
0.000121194
0.379901
-1.11053
75.4142
{-71.9437, -73.8658, -72.9873}
-116.606
1.49032

```

## Lander Trajectory



75.41417236881058`

75.4142

{-71.94371577934496`, -73.86577134593695`, -72.98731816193695`}

{-71.9437, -73.8658, -72.9873}

126.33

## 5 Orbiting in the coma

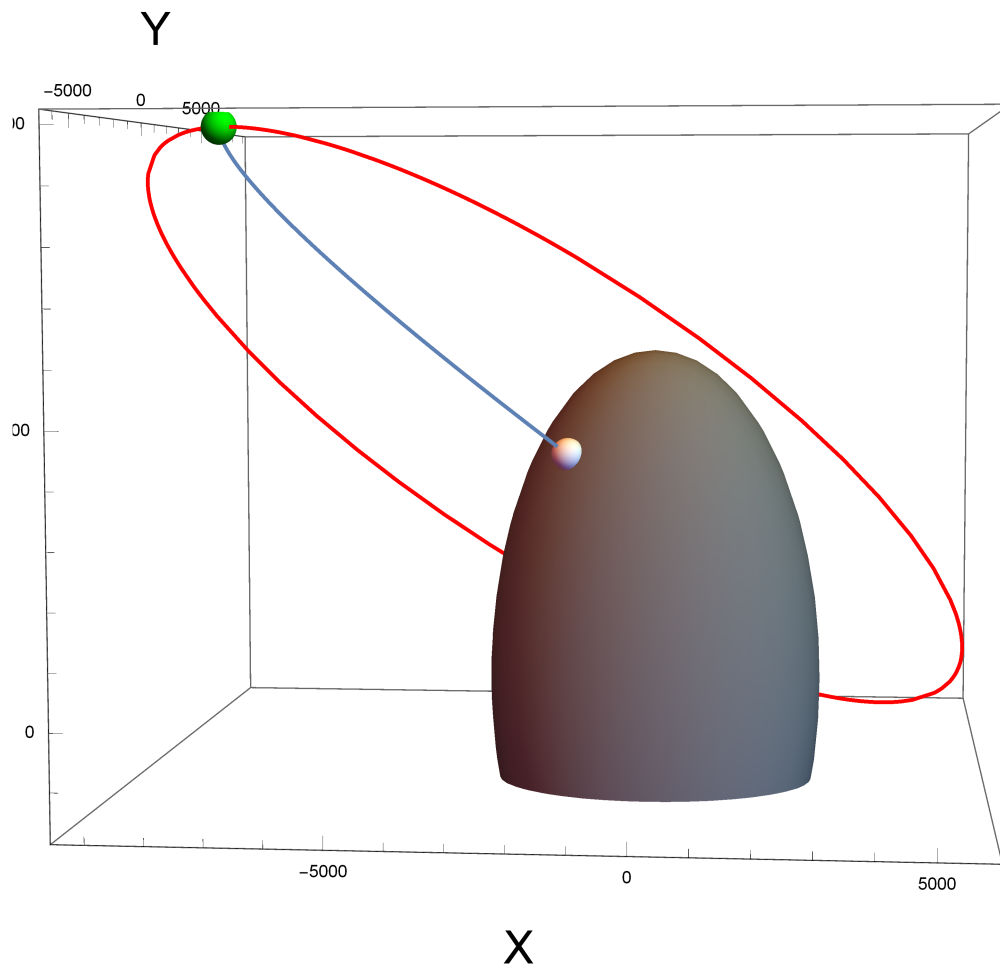
```

Clear[ro, Vo, Vd, x, y, z, To, F]
ro[t_] := {-1769.42 - 5230.58 Cos[0.0001 t] - 5362.31 Sin[0.0001 t],
  1233.56 - 6233.56 Cos[0.0001 t] + 4499.51 Sin[0.0001 t], 4187.62 + 5812.38 Cos[0.0001 t]};
rl[t_] := {-7000 + 0.001 t + 3 * 10-6 * t2, -5000 + 0.001 t + 2 * 10-6 * t2,
  10000 - 0.05 t - 2 * 10-6 * t2};
Lander = ParametricPlot3D[{rl[t]}, {t, 0, 50000},
  ImageSize → Large];
Orbiter =
  ParametricPlot3D[{ro[t]}, {t, 0,  $\frac{2\pi}{0.0001}$ }, PlotLabel → "Lander and Orbiter Trajectory",
  AxesLabel → {
    Style["X", Large],
    Style["Y", Large],
    Style["Z", Large]}, SphericalRegion → True,
  ImageSize → Large, PlotStyle → Red];
Comet = ContourPlot3D[ $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$ , {x, -a, a},
  {y, -b, b}, {z, -c, c}, PlotRange → All, BoxRatios → {a/c, b/c, 1},
  Mesh → None, ContourStyle → {Gray}, AxesLabel → {"x", "y", "z"}];
ReleaseLocation = Graphics3D[{Green, Sphere[rl[0], 300]}];
LandingLocation = Graphics3D[{White, Sphere[rl[42684.8], 300]}];
Show[Orbiter, ReleaseLocation, Comet, LandingLocation, Lander]

Vo[t_] := ro'[t];
Vd[t_] := 1.7 {E(-4*10-8) * (-1769.42 - 5230.58 Cos[0.0001 t] - 5362.31 Sin[0.0001 t])2,
  E(-4*10-8) * (1233.56 - 6233.56 Cos[0.0001 t] + 4499.51 Sin[0.0001 t])2, E(-1.6*10-9) * (4187.62 + 5812.38 Cos[0.0001 t])2};
To[t_] :=  $\frac{ro'[t]}{\sqrt{ro'[t].ro'[t]}}$ ;
F[t_] := Max[{0, ((Vo[t] - Vd[t]).To[t])}] * (2 * 10-6);
(*mass collected in a single orbit*)
Mc = (0.04) * NIntegrate[F[t], {t, 0,  $\frac{2\pi}{0.0001}$ }]

```

## Lander and Orbiter Trajectory



0.00417019